ENGR 1611 – Concepts I

Topic: Wheel, Screws, Gears
Thursday 3rd week

Outline

• Wheel and Axle
  • Mechanical advantage

• Screw
  • Mechanical advantage

• Gears
  • How do they work?
  • What is a gear ratio?
  • How does the gear ratio effect torque?
  • How does the gear ratio effect speed?
Wheel and Axle

• Moment balance

\[ \sum M_o = 0 \]

Moment = \( r \times F \)

CW moments = CCW moments

Wheel and Axle – Mech. Advantage

• Measure of the ability of a machine to amplify force
• Analogous to a lever

\[
\text{M.A.} = \frac{\text{Resistance (Force)}}{\text{Effort (Force)}}
\]

\[
\text{M.A.} = \frac{\text{Effort Arm}}{\text{Resistance Arm}}
\]
Exercise

- The diameter of the axle is 5”
- The handle moves through a diameter of 12”

How much force must be applied to lift a 75 lb bucket?

What is the mechanical advantage?

Screw

- Common applications – car jack, vice
- Screw pitch = distance between threads
Screw

Resistance

Effort

Screw - Mechanical Advantage

- Measure of the ability of a machine to amplify force

\[ M.A. = \frac{\text{Resistance (Force)}}{\text{Effort (Force)}} \]

\[ M.A. = \frac{\text{Distance Effort moves}}{\text{Distance Resistance moves}} = \frac{2\pi r}{p} \]

where \( p \) = pitch and \( r \) = radius
Exercise

• Your car has a flat tire.
• Your car weighs 1900 lbs.
• If you can exert a force of 75 lbs at a radius of 8 inches

What screw pitch on the jack is required to lift the car?

What is the mechanical advantage?

Gears

• Some examples include
  • Can opener
  • Cork screw
  • Transmission on your car
  • Bicycle

• Gears are used to
  • Change the direction of motion
  • Increase or decrease speed
  • Increase of decrease torque

• Gears are commonly used in power transmission applications because of their high efficiency (~98%)
Gears Configurations

• Spur gears
• Planetary and orbital gears
• Rack and pinion gears
• Bevel gears

Gear Ratio

• A gear will rotate with an angular velocity ($\omega$) with units of radians/second

• Gears have teeth that must mesh
  • Same pitch = same distance between teeth
  • There is a fixed ratio between the teeth and the gear radius

$$\frac{N_1}{N_2} = \frac{r_1}{r_2}$$

N = Number of teeth (defined as T in your reading)
**Gear Ratio - Velocity**

- Velocity of pitch point C on both bodies must be equal

\[ V_C = r_1 \cdot \omega_1 = r_2 \cdot \omega_2 \]

\[ \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} \]

\( \omega = \text{angular velocity (defined as } s \text{ in your reading)} \)

**Gear Ratio - Torque**

- Force of gear 1 on gear 2 is equal and opposite to force of gear 2 on gear 1

\[ F = \frac{T_1}{r_1} = \frac{T_2}{r_2} \]

\[ \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{T_1}{T_2} \]

\( \omega = \text{angular velocity (defined as } s \text{ in your reading)} \)
Gear Problems

- Master Equation
  \[ \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{T_1}{T_2} \]

- Small gear to large gear
  - Slower angular velocity, increased torque
- Large gear to small gear
  - Faster angular velocity, reduced torque

What are the gear ratios?

Let:
- \( r_{\text{green}} = 6 \) inches
- \( r_{\text{blue}} = 10 \) inches
- \( r_{\text{red}} = 15 \) inches
- \( \omega_{\text{green}} = 10 \) rad/sec

What is \( \omega_{\text{red}} \)?

Is \( T_{\text{red}} \) < or > \( T_{\text{green}} \)?